Eigenvalue Equations with Dressed Propagators in a Pole Regularized Spinor Theory *

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The massless Green's function in a pole regularized nonlinear spinor theory is dressed and the resulting eigenvalue equations are discussed. The coupling constant is more than doubled by this dressing, and the boson solutions change drastically. The old solutions disappear, a singlett deuteron solution with small binding energy appears. The fermion propagator is determined from a self-consistency requirement.

1. Introduction

A pole regularized nonlinear spinor theory has been proposed that allows for scattering calculations with the LSZ-technique. This pole theory can be used to test the functional scalar product of Stumpf's functional quantum theory [1, 2], comparing the S-matrix of this theory for nucleon-nucleon scattering with the LSZ-results. Approximations used so far in Heisenberg's nonlinear spinor theory with dipole regularization [3], and in the dynamically modified pole theory [4] are based on the φ-functional. The corresponding functional equation containes besides the fermion propagator F of the theory the massless Green's function G of the differential operator. To calculate S-matrix elements with the LSZ-reduction formula one has to dress G to the correct F, and it has been shown, how to achieve this dressing in the φ-equations [5]. The dressing is necessary for scattering calculations, but all eigenvalues shell show up as poles of S-matrix elements. Therefore one has to dress the Green's function G in all eigenvalue equations, too. In this paper the resulting equations are derived and solved in low approximations.

We use the notation of two earlier papers on the pole regularized spinor theory and refer to them as I [4] and II [5]. We need from the more general equations of I only the special case $e = 0$, $r = 1$. $r = 0$ was Heisenberg's dipole theory, that does not allow for the dressing of G, $e = 1$ was the momentum average discussed in functional quantum theory. It has been shown in II that one has to use the unsymmetrical φ-equations to achieve the dressing of G, but the equations, we will refer to, are the same for the integral average and without any symmetrization.

2. The Fermion Eigenvalue Equation

2.1. Derivation of the Equation with Dressed G

We cannot use the systematic approximation procedure of I (Capter 3.3), because this does not allow for the dressing of the Green's function G

$$-\bullet-:=G_{\alpha\beta}=i\int\frac{\mathrm{d}^4p}{(2\,\pi)^4}\frac{\overline{\Gamma}_{\alpha\beta}{}^{\nu}\,p_{\nu}}{[p^2+i\,\varepsilon]^2}\cdot\exp\{-i\,(p,x_\alpha-x_\beta)\} \quad (1)$$

to the propagator F, the vacuum expectation value of the time-ordered product of two field operators

$$-\bigcirc-:=F_{\alpha\beta}=\langle 0\,|\,T\,\psi_\alpha\,\psi_\beta\,|\,0\rangle=\int\mathrm{d}m^2\,\varrho\,(m^2)\,i$$
$$\cdot\int\frac{\mathrm{d}^4p}{(2\,\pi)^4}\frac{\overline{\Gamma}_{\alpha\beta}{}^{\nu}\,p_{\nu}}{[p^2+i\,\varepsilon]\,[p^2-m^2+i\,\varepsilon]}$$
$$\cdot\exp\{-i\,(p,x_\alpha-x_\beta)\}\,. \quad (2)$$

We have to use the unsymmetrical φ-equations, corresponding to the functional equation (I, 13)

$$\partial_\alpha\,|\,\Phi\,(j)\,\rangle=\{(\varrho_0\,G_{\alpha\beta}-F_{\alpha\beta})\,j_\beta$$
$$+i\,G_{\alpha\alpha'}\,V_{\alpha'\beta\gamma\delta}\,(d_\beta\,d_\gamma\,d_\delta+3\,F_{\beta\gamma}\,d_\delta)\}\,|\,\Phi\,(j)\,\rangle\,. \quad (3)$$

With the notation

$$\text{⊐ := } \varphi_{\alpha\beta\,\gamma} \qquad \times := \tfrac{1}{i}\,V$$
$$-\diamond- := -\bigcirc- - \varrho_0 -\bullet- \quad (4)$$

the lowest φ-equations for a fermion state are

$$\text{[diagram]} = \text{[diagram]} \tag{5}$$

$$\text{[diagram]} = \text{[diagram]} - 3\left(\text{[diagram]} - \text{[diagram]}\right)\text{[diagram]} + $$
$$+ 6\,\text{[diagram]} - \left(\text{[diagram]} - \text{[diagram]}\right)\text{[diagram]} \tag{6}$$

$$\text{[diagram]} = \text{[diagram]} + \cdots \tag{7}$$

The explicite form of (7) is given in a paper on Heisenberg's nonlinear spinor theory [Ref. 6, Equation (166)].

To dress the G-functions in these equations, we use the method developed in II. We have to insert the higher φ-functions so, that we produce G-chaines without branching. Therefore we have to insert (7) into (6) with the vertex in (6) attached to the G-line of (7). Thus we need from (7) only

$$\text{[diagram]} = \text{[diagram]} - 3\,\overline{(=-\times)}\text{[diagram]} - 6\,\text{[diagram]} + 12\,(=-\times)\text{[diagram]} + $$
$$+ 6\,\text{[diagram]} + 6\,\text{[diagram]} - 6\,(=-\times)\text{[diagram]} - 12\,\text{[diagram]} - 2\,\text{[diagram]} - \overline{(=-\times)}\text{[diagram]} \tag{8}$$

Inserting this into (6) we write down explicitly the three underlined terms of (8), calling the rest R

$$\text{[diagram]} = 6\,\text{[diagram]} + 6\,\text{[diagram]} - \left(\overline{=-\times}\right)\left(3\,\text{[diagram]} - \left[\text{[diagram]} - 6\,\text{[diagram]}\right]\text{[diagram]}\right) + \text{[diagram]}\,R \tag{9}$$

Besides the two terms containing $-\bigcirc\!\!\!)- = -\bigcirc- -\varrho_0 -\bullet-$, this equation has the form II (38) required for the dressing of G to F by the equation

$$(1 - -\bullet\square-)\; -\bigcirc- = \varrho_0 -\bullet- \tag{10}$$

with the lowest contribution to the mass term

$$-\square- = 6\,\text{[diagram]} \tag{11}$$

But with (11) the dressing equation can be written as

$$-\diamondsuit- = -\bigcirc- - \varrho_0 -\bullet- = 6\,\text{[diagram]} \tag{12}$$

and the wrong terms in (9) without a G-line at the upper leg disappear. Using (10), (11) we obtain from (9)

$$\text{[diagram]} = \tfrac{6}{\varrho_0}\,\text{[diagram]} - \tfrac{3}{\varrho_0}\,\overline{(=-\times)}\text{[diagram]} + \text{[diagram]}\,R \tag{13}$$

Finally we insert this result into (5). Neglecting all term containing higher φ-functions we arrive at the dressed eigenvalue equation

$$\text{[diagram]} = \tfrac{6}{\varrho_0}\,\text{[diagram]} \tag{14}$$

The remaining G-function in (14) is the one of the dressing Eq. (10) itself and cannot be dressed. In Chapter 2.3 we will use the dressing Eq. (10) with the mass term of (14) to calculate the fermion propagator F.

2.2. Solution of the Dressed Equation

The fermion eigenvalue Eq. (14) is explicitly

$$\varphi_a{}^F = \frac{6}{\varrho_0} \left(\frac{K^2}{2\,i} \right)^2 G_{\alpha\beta}\, V_{\beta\gamma}{}^i\, F_{\gamma\delta}\, V_{\delta\mu}{}^j\, F_{\mu\nu}\, V_{\nu\lambda}{}^i\, F_{\lambda\varrho}\, V_{\varrho\sigma}{}^j\, \varphi_\sigma{}^F . \tag{15}$$

We solve it in complete analogy to Eq. (46) of I and obtain

$$\left(1 + \frac{3}{2}\, \frac{1}{\varrho_0} \left(\frac{K}{2\,\pi} \right)^4 \mathcal{L}(I^2) \right) \varphi^F(I) = 0 \tag{16}$$

$$\mathcal{L}(I^2) := \frac{1}{(I^2)^2} \int d\alpha\, \varrho(\alpha)\, d\beta\, \varrho(\beta)\, d\gamma\, \varrho(\gamma)\; A(I^2;\,\alpha,\beta,\gamma) \tag{17}$$

$$A(I^2;\,\alpha,\beta,\gamma) = \frac{1}{\pi^4} \int \frac{d^4p\; d^4q\,(p,q)\,(I-p-q,I)}{p^2[p^2-\alpha]\,q^2[q^2-\beta]\,(I-p-q)^2[\,(I-p-q)^2-\gamma]} . \tag{18}$$

All the poles in A are understood with Feynman-$i\,\varepsilon$ (1), (2). We cannot solve this integral in terms of simple analytic functions, but we have to perform numerical integrations. Because of the poles in A this cannot be done directly. After introducing the usual Feynman parameter integrals, the momentum integrals can be done and we obtain

$$A(I^2;\,\alpha,\beta,\gamma) = I^2 \left(3 + I^2\, \frac{\partial}{\partial I^2} \right) \int\limits_0^1 \cdots \int\limits_0^1 dx\, dy\, dz\, du\, dv . \tag{19}$$

$$u^2(1-u)v(1-v)\,[I^2\,u(1-u)v(1-v)\; -\alpha\,x(1-u)v\; -\beta\,y(1-u)(1-v) -\gamma\,z\,u\,v(1-v)]^{-1} .$$

Only 3 of this 5 parameter integrals can be performed immediately and we would need a rather difficult twofold numerical integration

Using a generalized Thirring formula, we can write A as a spectral integral

$$A(I^2;\,\alpha,\beta,\gamma) = \int\limits_0^\infty d\mu^2\, \Omega(\alpha,\beta,\gamma;\mu)\; \frac{1}{I^2-\mu+i\,\varepsilon} . \tag{20}$$

But the spectral function Ω itself is an elliptic integral, given in the App. (A 26), and again we have to perform two integrals numerically.

It is possible, however, to transform $A(s;\,\alpha,\beta,\gamma)$ to a single integral over $A(s;\,\alpha,\beta,0)$. This A^{II} with only two nonzero masses is related to the convolution $F*F$ of two propagators F alone (App. 3) and can be evaluated analytically. In App. 1 we derive from (19) the following representation for A

$$A(s;\,\alpha,\beta,\gamma) = s \int\limits_0^1 du \left\{ s\, \frac{1-u^2}{2\,\gamma}\, b(u\,s) + \left(u + \frac{1}{2}\; -s\, \frac{1-u^2}{2\,\gamma} \right) b\left(u\,s - \frac{u}{1-u}\,\gamma \right) \right\} \tag{21}$$

$$A^{II}(s;\,\alpha,\beta) := A(s;\,\alpha,\beta,0)\; = s\cdot b(s;\,\alpha,\beta)$$

$$= \frac{1}{3} + \frac{s^2}{6\,\alpha\beta}\, \ln\left(\frac{-s}{\sqrt{\alpha\beta}} \right) + \left[\frac{s(\alpha-\beta)}{2\,\alpha\beta} - \frac{\alpha^2-\beta^2}{2\,\alpha\beta} + \frac{(\alpha-\beta)^3}{6\,s\,\alpha\beta} \right] \ln\sqrt{\frac{\alpha}{\beta}} \tag{22}$$

$$+ \frac{(\alpha-s)^3}{6\,s\,\alpha\beta}\, \ln\left(1 - \frac{s}{\alpha} \right) + \frac{(\beta-s)^3}{6\,s\,\alpha\beta}\, \ln\left(1 - \frac{s}{\beta} \right) - \frac{1}{12\,s\,\alpha\beta}\, \Lambda(s,\alpha,\beta)^{3/2}\, \ln\frac{s-\alpha-\beta-\sqrt{\Lambda(s,\alpha,\beta)}}{s-\alpha-\beta+\sqrt{\Lambda(s,\alpha,\beta)}}$$

$$\Lambda(s,\alpha,\beta) := s^2 + \alpha^2 + \beta^2 - 2\,s\,\alpha - 2\,s\,\beta - 2\,\alpha\,\beta .$$

The numerical integration of (21) is still not trivial, because the integrand is not a regular function. A simple gaussian integration does not converge, for example, if the number of meshpoints is increased. But the thresholds of A^{II} are known, either from the explicit formula (22) or from the fact that A^{II}

is related to the convolution $F * F$: they occur at $s = 0$, $(\sqrt{\alpha} \pm \sqrt{\beta})^2$. Inbetween A^{II} or b is infinitely often differentiable on the real axis and a gaussian integration converges rapidly. For more details see appendix 1, especially Table 1.

As in I we use the fermion eigenvalue Eq. (15) to determine the coupling constant of the theory. We assume that the fermion propagator F is dominated by the pole at the lowest fermion eigensolution. We make an ansatz for the spectral function (2)

$$\varrho(m^2) = Z \delta(m^2 - \varkappa^2) + \varrho^{\varDelta}(m^2) \qquad (23)$$

with $\varrho^{\varDelta}(m^2)$ finite at the pole $m^2 = \varkappa^2$. In evaluating the fermion function $\mathcal{L}(I^2)$ (17) we keep only the pole term of F. This approximation will be justified later (Capter 2.3), when we discuss the selfconsistence of the pole ansatz (23). Neglecting $\varrho^{\varDelta}$ the integrals in (17) are trivial and we obtain

$$\mathcal{L}(I^2) = \frac{Z^3}{(I^2)^2} \, A(I^2; \varkappa^2, \varkappa^2, \varkappa^2) \; . \qquad (24)$$

The eigenvalue equation (15) becomes finally

$$\left(1 + \frac{3}{2} \frac{Z^3}{\varrho_0} \left(\frac{K}{2 \pi \varkappa}\right)^4 L^{\mathrm{III}}(\lambda)\right) \varphi^{\mathrm{F}}(\lambda) = 0 \qquad (25)$$

with

$$L^{\mathrm{III}}(\lambda) := (1/\lambda^2)(A(I^2; \varkappa^2, \varkappa^2, \varkappa^2) \quad \lambda := I^2/\varkappa^2 . \qquad (26)$$

This fermion function L^{III} is shown in Fig. 1, compared with L^{II}, the fermion function $G * F * F$ of I, there denoted by L. Some values of $\lambda \cdot L^{\mathrm{III}}(\lambda)$ are given in Tab. 2 in the appendix.

Equation (25) shall have its solution at $\lambda = 1$, thus we obtain an equation for the coupling constant
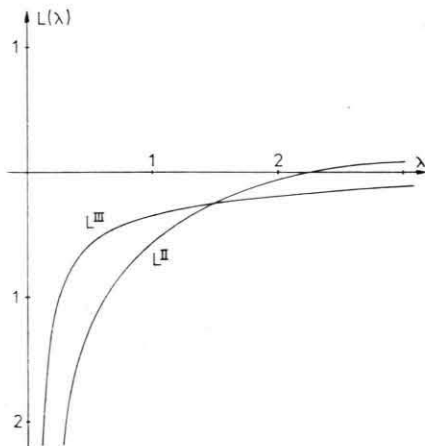
Fig. 1. Fermion function $L^{\mathrm{III}}(\lambda \sim F * F * F$ with three dressed propagators compared with $L^{\mathrm{II}}(\lambda) \sim F * F * G$ with one bare propagator.

$$\left(\frac{Z^3}{\varrho_0} \frac{K}{2 \pi \varkappa}\right)_4 = \frac{-2}{3} \frac{1}{L^{\mathrm{III}}(1)} = 1.941 . \qquad (27)$$

This special value $L^{\mathrm{III}}(1)$ can be calculated explicitly because at the thresholds of A the elliptic integral for the spectral function Ω (20) degenerates. The integration yields some dilogarithms and one obtains finally

$$L^{\mathrm{III}}(1) = \frac{\pi^2}{4} + \frac{1}{4} - \frac{9\sqrt{3}}{16} \pi = -0.3434 . \qquad (28)$$

For the calculation of boson eigenvalues we will need the physical coupling constant $K^2_{\mathrm{phys}} \lambda^2$, that determines the nucleon-nucleon scattering in the lowest approximation (II.56)

$$\left(\frac{K_{\mathrm{phys}}}{2 \pi \varkappa}\right)^2 = \frac{Z^2}{\varrho_0} \left(\frac{K}{2 n \varkappa}\right)^2 . \qquad (29)$$

Besides (27) we therefore need the ratio Z/ϱ_0.

To determine Z/ϱ_0 we have to calculate the fermion propagator out of the theory.

2.3. Selfconsistent Calculation of the Fermion Propagator

In analogy to the derivation of the fermion eigenvalue Eq. (14) we can derive an equation for the fermion propagator F with the G-function dressed

$$\text{—○— } = \varrho_0 \text{ —●— } + \frac{1}{\varrho_0} \text{ —◈○—} \qquad (30)$$

This dressed mass operator $F * F * F$ appeared already in higher approximation of the boson equations in II (54). (30) is a nonlinear equation for F

$$F_{\alpha\tau} = \varrho_0 \, G_{\alpha\tau} + \frac{6}{\varrho_0} \left(\frac{K^2}{2 i}\right)^2$$
$$\cdot \, G_{\alpha\beta} V_{\beta\gamma}{}^{\mathrm{i}} F_{\gamma\delta} V_{\delta\mu}{}^{\mathrm{j}} F_{\mu\nu} \, V_{\nu\lambda}{}^{\mathrm{i}} F_{\lambda\varrho} V_{\varrho\sigma}{}^{\mathrm{j}} F_{\sigma\tau} . \qquad (31)$$

We cannot solve it exactly, but use the same approximation method as in I or II, that can be justified even better in our dressed case. We solve (30) or (34) formally for F and obtain

$$\text{—○— } = \varrho_0 \text{ —●— } \left[1 - \frac{1}{\varrho_0} \text{ ●} \text{◈}\right]^{-1} \qquad (32)$$

or with the notations (17), (18)

$$F_{\alpha\beta} = \varrho_0 \, G_{\alpha\beta} \left[1 + \frac{3}{2} \frac{1}{\varrho_0} \left(\frac{K}{2 \pi}\right)^4 \mathcal{L}(I^2)\right]^{-1} . \qquad (33)$$

Again we calculate $\mathcal{L}$ with the pole term of F only and compare the right hand side of (33) with the ansatz

$$F_{\alpha\beta}{}^{\mathrm{a}}(I) = i \frac{\overline{\Gamma}_{\alpha\beta}{}^{\nu} I_\nu Z}{I^2(I^2 - \varkappa^2)} = : \frac{i \, \overline{\Gamma}_{\alpha\beta}{}^{\nu} I_\nu Z}{I^2 \varkappa^2} \cdot N^{\mathrm{a}}(\lambda) . \qquad (34)$$

We denote the calculated F by F^c

$$F_{\alpha\beta}{}^c(I) = i\frac{\overline{\Gamma}_{\alpha\beta}{}^\nu I_\nu Z}{I^2 \varkappa^2} N^c(\lambda) . \tag{35}$$

From Fig. 1 we learn that F or N has a pole at a finite I^2. This pole should be at $I^2 = \varkappa^2$ (or $\lambda = 1$), fixing the coupling constant to the value given in (27). The residuum at this pole of N should be 1, this fixes the renormalization constant Z to the value

$$\frac{Z}{\varrho_0} = L^{\mathrm{III}}(1) \cdot \left[\left.\frac{\partial L^{\mathrm{III}}}{\partial\lambda}\right|_{\lambda=1}\right]^{-1} = 0.696^{-1} = 1.437 . \tag{36}$$

Inserting (24), (27) and (3) into (33), we obtain

$$N^c(\lambda) = \frac{L^{\mathrm{III}\prime}(\lambda)}{\lambda\left(L^{\mathrm{III}}(\lambda) - L^{\mathrm{III}}(1)\right)} . \tag{37}$$

This resulting N^c is shown in Fig. 2 together with the ansatz $N^a = (\lambda - 1)^{-1}$ and the difference N^Δ. N^Δ is rather small for all λ, especially for $\lambda < 2$, where the coupling constant has been determined. Therefore the neglection of N^Δ is even better justified than in the case $F * F * G$ discussed in II, where the dipole of G causes a bigger N^Δ (II, Figure 1).

Thus the calculated right hand side of (33) is in good agreement with the pole ansatz.

F^c has two poles, at $I^2 = 0$ with residuum $1.033\,Z$ and at $I^2 = \varkappa^2$ with residuum $1 \cdot Z$. For $I^2 \to \infty$ the asymptotic behavior is correctly $\sim (I^2)^{-2}$ with $F^c/F^a \to \varrho_0/Z$. Z/ϱ_0 should be less than 1 an a
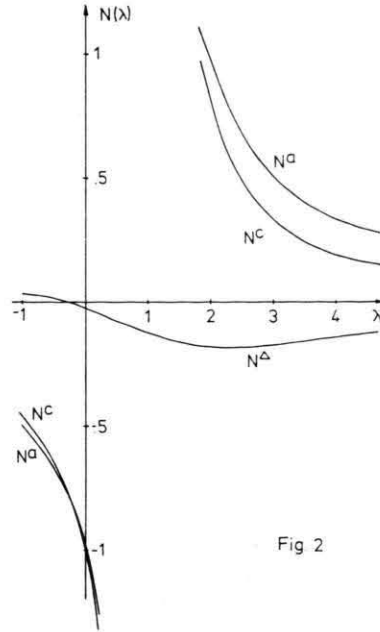
Fig. 2. Calculated pole function $N^c(\lambda)$ of the fermion propagator compared with the ansatz $N^a(\lambda) = (\lambda - 1)^{-1}$. The difference $N^\Delta(\lambda)$ is rather small, justifying its neglection in the convolution integral A.

canonical theory. If Z/ϱ_0 will remain bigger than one in higher approximations, one would have a more complicated indefinite metric that cannot be described with the simple regularization pole. The $\varrho(m^2)$ in F (2) could not be positive, and the formal canonical theory described in the appendix of I would still have indefinite metric.

3. The Boson Eigenvalue Equation

The lowest approximation for the boson eigenvalue equation with dressed Green's function has been derived in II to be

$$\exists = -\tfrac{3}{\varrho_0} \text{⋈} \tag{38}$$

or written down explicitely with the notation of I (57)

$$\varphi_{\alpha\beta}{}^{\mathrm{B}} = -\frac{3}{\varrho_0}\left(\frac{K^2}{2i}\right) F_{\alpha\beta'} V_{\beta'\gamma}{}^{\mathrm{i}} F_{\gamma\beta} V_{\mu\nu}{}^{\mathrm{j}} \varphi_{\nu\mu}{}^{\mathrm{B}} . \tag{39}$$

We solve it in analogy to Eq. I (57) and obtain with the projection operator $P_{\mathrm{SI}}^{\mathrm{B}}$ for baryon number B, spin S and isospin I

$$\varphi^{\mathrm{B}}(I) = -\frac{1}{\varrho_0}\left(\frac{K}{2\pi\varkappa}\right)^2 \frac{1}{4}\left[(3 P_{00}{}^0 + P_{01}{}^0) q_0 + (3 P_{10}{}^0 + P_{11}{}^0) q_1 - 2 P_{01}{}^2 q_2\right] \varphi^{\mathrm{B}} . \tag{40}$$

The boson functions q_i are defined by

$$q_i(\lambda) := \int \mathrm{d}\alpha\, \varrho(\alpha) \int \mathrm{d}\beta\, \varrho(\beta)\, Q_i(I^2; \alpha, \beta) \tag{41}$$

$$\left(\frac{1}{2}\,g^{\mu\nu} - \frac{I^\mu I^\nu}{I^2}\right)Q_0 + \left(\frac{1}{2}\,g^{\mu\nu} + \frac{I^\mu I^\nu}{I^2}\right)Q_1 := \frac{4\,i\,\sqrt{\alpha\,\beta}}{\pi^2}\int d^4p\,\frac{p^\mu(I-p)^\nu}{p^2[p^2-\alpha]\,(I-p)^2\,[(I-p)^2-\beta]} \tag{42}$$

$$Q_2 = \tfrac{1}{2}\,Q_0 + \tfrac{3}{2}\,Q_1\,. \tag{43}$$

The Q_i-functions for different masses are given in appendix 2. We will take the pole term of the ansatz (23) and obtain

$$q_i(\lambda) = Z^2\,Q_i(I^2;\varkappa^2,\varkappa^2) \qquad \lambda = \frac{I^2}{\varkappa^2}\,. \tag{44}$$

This Q_i-functions for equal masses are

$$Q_0(\lambda) = \frac{2}{\lambda} + 2\,\frac{(1-\lambda)^2}{\lambda^2}\,\ln|1-\lambda| + \frac{1}{\lambda}\,\sqrt{\lambda(\lambda-4)}\,\ln\frac{\lambda-2-\sqrt{\lambda(\lambda-4)}}{\lambda-2+\sqrt{\lambda(\lambda-4)}}\,, \tag{45}$$

$$Q_1(\lambda) = -\frac{2}{3\,\lambda} + \frac{2}{3}\,\lambda\ln|\lambda| - 2\,\frac{(1+2\,\lambda)(1-\lambda)^2}{3\,\lambda^2}\,\ln|1-\lambda| + \frac{1-\lambda}{3\,\lambda}\,\sqrt{}\ln\div\,, \tag{46}$$

$$Q_2(\lambda) = \lambda\ln|\lambda| - 2\,\frac{(1-\lambda)^2}{\lambda}\,\ln|1-\lambda| + \frac{2-\lambda}{2\,\lambda}\,\sqrt{}\ln\div\,. \tag{47}$$

The absolute values in this functions are obtained, because we suppressed the imaginary parts from the regularization poles of F. All three functions have imaginary parts, starting at the two-nucleon threshold $\lambda = 4$, for we have

$$\mathrm{Im}\,\sqrt{\lambda(\lambda-4)}\,\ln\frac{\lambda-2-\sqrt{\lambda(\lambda-4)}}{\lambda-2+\sqrt{\lambda(\lambda-4)}} = \Theta(\lambda-4)\,\sqrt{\lambda(\lambda-4)}\,\cdot 2\,\pi\,. \tag{48}$$

The boson functions Q_i are shown in Figure 3.

To solve the eigenvalue equation (40) we need the physical coupling constant (29) and obtain with (27) and (36)

$$\left(\frac{K_{\mathrm{phys}}}{2\,\pi\,\varkappa}\right)^2 := \frac{Z^2}{\varrho_0}\left(\frac{K}{2\,\pi\,\varkappa}\right)^2 = \left[\frac{3}{2}\,\frac{\partial L^{\mathrm{III}}}{\partial\lambda}\bigg|_{\lambda=1}\right]^{-\frac{1}{2}}$$

$$= 1.670 \qquad\qquad = 0.599^{-1}\,. \tag{49}$$

Without the dressing of G to F this physical coupling constant is 0.760 (II.59) and the dressing has doubled it.

The boson eigensolutions are more drastically changed. We get no solution for $B = 0$ at all. The π- and η-solution of I disappear after the dressing. The reason is the following: The bare boson functions q^{I} with only one dressed propagator are dominated for $|\lambda| \ll 1$ by the logarithmic singularity at $\lambda = 0$, that stems from the dipole of the Green's function G (1). This dipole disappears after the dressing and the Q_i-functions are regular at $\lambda = 0$. Therefore it is possible, that the meson solutions of I are no genuine solutions of the pole spinor theory, but stem only from the approximation procedure involving the massless G-function. Higher approximations, both with bare and dressed G-functions, are necessary to clarify the situation. They involve
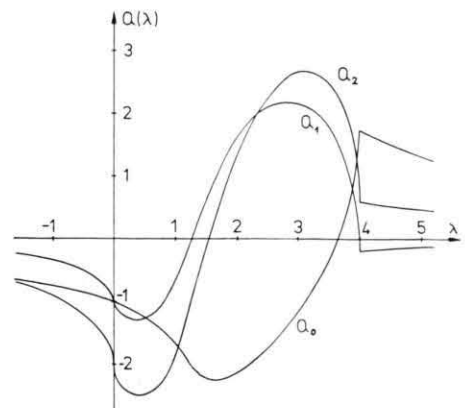

Fig. 3. Boson functions $Q_i \sim F \ast F$ with two dressed propagators.

a nonlocal interaction and one has to solve a spinor Bethe-Salpeter equation using variational methods [7].

This has been done recently in the dipole regularized form of the spinor theory [8], the pole theory should yield no additional problems.

For baryon number 2 Eq. (40) has a solution. In our low approximation of the φ-equation appears only P_{01}^2, that is a spin singlett, isospin triplett state. Thus we cannot obtain the physical spin-triplett deuteron d^3, but the spin singlett deuteron d^1, that is looked upon as an anti-bound state on the second sheet at 65 KeV. Inserting (44) and (49) into (40), we determine the mass eigenvalue D for this d^1 state out of the equation

$$\left(1 - \frac{1}{2}\frac{Z^2}{\varrho_0}\left(\frac{K}{2\pi\varkappa}\right)^2 q_2\left(\frac{D^2}{\varkappa^2}\right)\right)P_{01}{}^2\,\varphi^{\mathrm{B}} = 0\,, \quad (50)$$

$$q_2\left(\frac{D^2}{\varkappa^2}\right) = 2\left(\frac{K_{\mathrm{phys}}}{2\pi\varkappa}\right)^{-2} = 1.1974\,, \quad (51)$$

$$D^2 = 3.9638\,\varkappa^2\,. \quad (52)$$

From (52) we calculate the binding energy E

$$E := D - 2\,\varkappa = 0.00907\,\varkappa = 8.52\;\mathrm{MeV}\,. \quad (53)$$

Thus we got a singlett deuteron with small binding energy.

Appendix

1. Calculation of the Dressed Fermion Integral

The integral in question is with $s = I^2$

$$A(s;\alpha,\beta,\gamma) := \frac{1}{\pi^4}\int\frac{\mathrm{d}^4p\,\mathrm{d}^4q\,(p,q)\,(I-p-q,I)}{p^2[p^2-\alpha]\,q^2[q^2-\beta]\,(I-p-q)^2\,[(I-p-q)^2-\gamma]}\,. \quad (A.1)$$

We want to prove (21), that A may be expressed as an integral over the same function with one zero mass.
For the more general integral

$$\begin{aligned}
A_{\lambda\mu\nu} &:= \frac{1}{\pi^4}\int\frac{\mathrm{d}^4p\,\mathrm{d}^4q\,p_\lambda\,q_\mu\,(I-p-q)_\nu}{p^2[p^2-\alpha]\,q^2[q^2-\beta]\,(I-p-q)^2\,[(I-p-q)^2-\gamma]}\\
&= A_1\left(g_{\lambda\mu}\frac{I_\nu}{I^2} + g_{\mu\nu}\frac{I_\lambda}{I^2} + g_{\nu\lambda}\frac{I_\mu}{I^2}\right) + A_2\,\frac{I_\lambda I_\mu I_\nu}{[I^2]^2}
\end{aligned} \quad (A.2)$$

one derives with the usual Feynman-parameters (I appendix)

$$A_1(s;\alpha,\beta,\gamma) = \frac{s}{2}\int_0^1\frac{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}u\,\mathrm{d}v\,u^2(1-u)\,v(1-v)}{s\,u(1-u)\,v(1-v) - \alpha\,x(1-u)\,v - \beta\,y(1-u)\,(1-v) - \gamma\,z\,u\,v(1-v)}\,,$$

$$A_2(s;\alpha,\beta,\gamma) = 2\,s^2\frac{\partial}{\partial s}\frac{1}{s}\,A_1(s;\alpha,\beta,\gamma)\,. \quad (A.3)$$

From (A.2) (A.3) we get the integral A back as

$$A(s;\alpha,\beta,\gamma) = g^{\lambda\mu}\,I^\nu\,A_{\lambda\mu\nu} = 6\,A_1 + A_2 = 2\,s\left(3 + s\frac{\partial}{\partial s}\right)\frac{1}{s}\,A_1(s;\alpha,\beta,\gamma)\,. \quad (A.4)$$

All this formulae are valid for $\gamma = 0$, too. The corresponding functions with only two nonzero masses are denoted by

$$A^{\mathrm{II}}(s) \equiv A^{\mathrm{II}}(s;\alpha,\beta) := A(s;\alpha,\beta,0)\,. \quad (A.5)$$

For $\gamma = 0$ two of the five integrals in $A_1{}^{\mathrm{II}}$ are performed easily. Let us rewrite Eq. (A.3) as

$$A_1(s;\alpha,\beta,\gamma) = \frac{s}{2}\int_0^1\mathrm{d}z\int_0^1\mathrm{d}u\,u^2\,b\left(u\,s - z\,\frac{u}{1-u}\,\gamma;\alpha,\beta\right),$$

$$b(s) \equiv b(s;\alpha,\beta) := \int_0^1\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}v\,\frac{-v(1-v)}{\alpha\,x\,v + \beta\,y(1-v) - s\,v(1-v)} \quad (A.6)$$

then we obtain for $\gamma = 0$

$$A_1{}^{\mathrm{II}}(s) = \frac{s}{2} \int\limits_0^1 dz \int\limits_0^1 du\, u^2\, b(u\,s) = \frac{1}{2\,s^2} \int\limits_0^s dt\, t^2\, b(t) \tag{A.7}$$

and with (A.4)

$$A^{\mathrm{II}}(s; a, \beta) = 2\,s\left(3 + s\,\frac{\partial}{\partial s}\right)\frac{1}{s}\, A_1{}^{\mathrm{II}}(s; a, \beta) = s\cdot b(s; a, \beta)\,. \tag{A.8}$$

Now we have expressed A_1 as a twofold integral over A^{II}. Inserting this A_1 into (A.4) we obtain

$$A(s; a, \beta, \gamma) = s\left(3 + s\,\frac{\partial}{\partial s}\right)\int\limits_0^1 dz \int\limits_0^1 du\, u^2\, b\left(u\,s - z\,\frac{u}{1-u}\,\gamma\right) \tag{A.9}$$

We denote the derivative resp. the integral of $b(s)$ by

$$b'(s) := \frac{\partial}{\partial s}\, b(s; a, \beta)\,, \qquad \widehat{b}(s) := \int\limits_0^s ds'\, b(s'; a, \beta)\,, \tag{A.10}$$

and obtain from (A.9)

$$A(s; a, \beta, \gamma) = s\int\limits_0^1 dz \int\limits_0^1 du\left[3\,u^2\, b\left(u\,s - z\,\frac{n}{1-u}\,\gamma\right) + s\,u^2\, b'(\ldots)\right]$$

$$= \frac{s}{\gamma}\int\limits_0^1 du\left[3\,u(1-u)\,\widehat{b}\left(u\,s - z\,\frac{u}{1-u}\,\gamma\right) + s\,u^2(1-u)\,b(\ldots)\right]_{z=1}^{z=0}\,. \tag{A.11}$$

Partial integration yields no surface terms, because one can derive from (A.10) ($\widehat{b}$ can be calculated explicitly, too)

$$\lim_{u\to 1}(1-u)\,\widehat{b}\left(-\frac{\gamma}{1-u}\right) = 0\,, \quad \widehat{b}(0) = 0\,.$$

Therefore we obtain from (A.11)

$$A(s; a, \beta, \gamma) = s\int\limits_0^1 du\left\{s\,\frac{1-u^2}{2\,\gamma}\,b(u\,s) + \left(u + \frac{1}{2} - s\,\frac{1-u^2}{2\,\gamma}\right)b\left(u\,s - \frac{u}{1-u}\,\gamma\right)\right\}\,. \tag{A.12}$$

This integral over u has been performed numerically. The three integrals of (A.6) can been done without principal difficulties. For $s < 0$ the integrand is regular, and the physical value $s > 0$ is reached by analytic continuation, replacing s by $s + i\,\varepsilon$.

Performing the three integrations in (A.6) we obtain for b

$$b(s; a, \beta) = \frac{1}{3\,s} + \frac{s}{6\,a\,\beta}\ln\left(\frac{-s}{\sqrt{a\,\beta}}\right) + \left[\frac{a-\beta}{2\,a\,\beta} - \frac{a^2-\beta^2}{2\,a\,\beta\,s} + \frac{(a-\beta)^3}{6\,a\,\beta\,s^2}\right]\ln\sqrt{\frac{a}{\beta}} \tag{A.13}$$

$$+ \frac{(a-s)^3}{6\,a\,\beta\,s^2}\ln\left(1 - \frac{s}{a}\right) + \frac{(\beta-s)^3}{6\,a\,\beta\,s^2}\ln\left(1 - \frac{s}{\beta}\right) - \frac{1}{12\,a\,\beta\,s}\,\Lambda(s, a, \beta)\,R(s; a, \beta)\,,$$

$$R(s; a, \beta) := \sqrt{\Lambda(s, a, \beta)}\ln\frac{s - a - \beta - \sqrt{\Lambda(s, a, \beta)}}{s - a - \beta + \sqrt{\Lambda(s, a, \beta)}}\,,$$

$$\Lambda(s, a, \beta) := s^2 + a^2 + \beta^2 - 2\,s\,a - 2\,s\,\beta - 2\,a\,\beta\,. \tag{A.14}$$

For real s we can rewrite $R(s; a, \beta)$ as

$$R(s; a^2, b^2) = 4\sqrt{\Lambda(s; a^2, b^2)}\left\{\Theta\left((a-b)^2 - s\right)\mathrm{Arsh}\sqrt{\frac{(a-b)^2 - s}{4\,a\,b}}\right. \tag{A.15}$$

$$\left. - \Theta\left(-\Lambda(s, a^2, b^2)\right)\arcsin\sqrt{\frac{s - (a-b)^2}{4\,a\,b}} - \Theta\left(s - (a+b)^2\right)\left[\mathrm{Arcoh}\sqrt{\frac{s - (a-b)^2}{4\,a\,b}} + i\,\frac{\pi}{2}\right]\right\}\,.$$

For equal masses $\alpha = \beta = \varkappa^2$ we obtain of course the result of I (65) with $\lambda = \dfrac{l^2}{\varkappa^2}$

$$R_1(\lambda) = \frac{1}{\varkappa^2} R(l^2; \varkappa^2, \varkappa^2) \qquad L(\lambda) = \frac{1}{\lambda^2} A^{\mathrm{II}}(l^2, \varkappa^2, \varkappa^2) . \tag{A.16}$$

For checks of the numerical calculations it is useful, to have explicit formulae for the functions b, b', $\hat{b}$ (A.10). For equal masses they are

$$\varkappa^2 b(\lambda) = \frac{1}{3\,\lambda} + \frac{\lambda}{6} \ln(-\lambda) + \frac{(1-\lambda)^3}{3\,\lambda^2} \ln(1-\lambda) + \frac{4-\lambda}{12\,\lambda} R_1(\lambda) , \tag{A.17}$$

$$\varkappa^4 b'(\lambda) = -\frac{2}{3\,\lambda^2} + \frac{1}{6} \ln(-\lambda) - \frac{2+\lambda}{3\,\lambda^3} (1-\lambda)^2 - \frac{2+\lambda}{12\,\lambda^2} R_1(\lambda) , \tag{A.18}$$

$$\hat{b}(\lambda) = -\frac{1}{3} + \frac{\lambda^2}{12} \ln(-\lambda) - \frac{1}{6\,\lambda} (2+5\,\lambda-\lambda^2)(1-\lambda)\ln(1-\lambda)$$

$$+ \frac{10-\lambda}{24} R_1(\lambda) + \frac{1}{8} \ln^2 \frac{\lambda-2-\sqrt{\lambda(\lambda-4)}}{\lambda-2+\sqrt{\lambda(\lambda-4)}} - \int_0^\lambda \frac{d\lambda^1}{\lambda^1} \ln(1-\lambda^1) . \tag{A.19}$$

The last term is a dilogarithm, expansions of which are given in I (A.36), (A.36). For a sufficient accuracy in the numerical computation of these functions, one needs their expansions for big and small λ, for in this limits those functions are small differences of diverging quantities. One obtains for $|\lambda| < 1$

$$\varkappa^2 b(\lambda) = -\frac{1}{2} + \frac{\lambda}{6} \ln\lambda - \frac{\lambda}{6} + \sum_{n=2}^\infty \lambda^n \left[2\,\frac{(n-2)!}{(n+2)!} - \frac{n!\,(n-2)!}{(2\,n+1)!} \right] , \tag{A.20}$$

$$\varkappa^4 b'(\lambda) = \frac{1}{6} \ln\lambda + \frac{2}{15}\,\lambda + \frac{13}{280}\,\lambda^2 + \frac{3}{160}\,\lambda^3 + \dots , \tag{A.21}$$

$$\hat{b}(\lambda) = -\frac{1}{2} + \frac{\lambda^2}{12} \ln\lambda - \frac{\lambda^2}{8} + \frac{1}{45}\,\lambda^3 + \dots \tag{A.22}$$

and for $|\lambda| \gg 1$

$$\varkappa^2 b(\lambda) = \frac{1}{\lambda} + \frac{\ln\lambda}{\lambda^2} + \frac{1}{2\,\lambda^2}\,\frac{\ln\lambda}{\lambda^3} + \dots , \tag{A.23}$$

$$\varkappa^4 b'(\lambda) = -\frac{1}{\lambda^2} - 2\,\frac{\ln\lambda}{\lambda^3} - 3\,\frac{\ln\lambda}{\lambda^4} + \dots , \tag{A.24}$$

$$\hat{b}(\lambda) = \frac{1}{2} - \frac{\pi^2}{6} + \ln\lambda - \frac{\ln\lambda}{\lambda} - \frac{3}{2\,\lambda} - \frac{\ln\lambda}{2\,\lambda^2} + \dots \tag{A.25}$$

The numerical integration of (A.12) is done by the gaussian integration formula [9]. After subdividing the integration interval into parts corresponding to the thresholds of b or A^{II} $\left(s=0, (\sqrt{\alpha}\pm\sqrt{\beta})^2\right)$, the approximations converge rapidly with growing number n of meshpoints. An example is shown in Table 1. Some values of the integral A for equal masses are listed in Table 2.

The analytic properties of the integral A (A.1) are most easily seen, if one rewrites it in a spectral representation, using a generalized Thirring formula.

$$A(s; \alpha, \beta, \gamma) = \int_0^\infty \frac{d\nu}{s-\nu+i\,\varepsilon}\,\Omega(\nu; \alpha, \beta, \gamma) ,$$

$$\Omega(\nu; \alpha, \beta, \gamma) := \frac{1}{\alpha\,\beta\,\gamma} \left[\omega(\nu; \alpha, \beta, \gamma) - \omega(\nu; 0, 0, 0) \right. \tag{A.26}$$

$$\left. - \omega(\nu; \alpha, \beta, 0) - \omega(\nu; \alpha, 0, \gamma) - \omega(\nu; 0, \beta, \gamma) + \omega(\nu; \alpha, 0, 0) + \omega(\nu; 0, \beta, 0) + \omega(\nu; 0, 0, \beta) \right] ,$$

n	$A(\varkappa^2; \varkappa^2, \varkappa^2, \varkappa^2)$
2	-0.3445487
4	-0.3434932
8	-0.3433907
16	-0.3433856
exact	-0.3433853

Table 1. Convergence of the numerical integration in Eq. (21) after dividing the integral into parts, according to the thresholds of the integrand, n is the number of gaussian meshpoints.

λ	$\lambda L^{III}(\lambda)$
$-\infty$	$\sim \lambda^{-1}$
-1	-0.1748
-0.5	-0.1976
-0.2	-0.2157
0	-0.2313
0.2	-0.2517
0.5	-0.2864
1.0	-0.3434
1.5	-0.3846
2	-0.3926
4	-0.0911
9	$+0.1924$
$+\infty$	$\sim \lambda^{-1}$

Table 2. Some values of the fermion function for equal masses
$(1/\lambda) A(s; \mathfrak{a}, \mathfrak{a}, \mathfrak{a}) \equiv \lambda L^{III}(\lambda)$.

$$\omega(\nu; a, \beta, \gamma) := \Theta\left(\nu - (\sqrt{a} + \sqrt{\beta} + \sqrt{\gamma})^2\right)$$

$$\cdot \frac{-1}{4\nu} \int_{x_3}^{x_2} \frac{dx}{x} \sqrt{(x_1-x)(x_2-x)(x-x_3)(x-x_4)} \left(\frac{x_1+x_2}{2} - x\right)\left(x - \frac{x_3+x_4}{2}\right), \quad (A.27)$$

$$x_{1,2} = (\sqrt{\nu} \pm \sqrt{\beta})^2, \quad x_{3,4} = (\sqrt{a} \pm \sqrt{\gamma})^2.$$

The elliptic integral (A.27) is symmetric in a, β, γ.

To calculate A without the two factors in the numerator, one simply has to omit the last two factors in (A.27). The representation (A.26) is derived with the general Thirring formula

$$\int_{(\sqrt{a}+\sqrt{\beta})^2}^{\infty} d^4q \, \frac{(p,p-q)^k (p-q)^{2l} (p,q)^m q^{2n}}{[(p-q)^2 - a + i\varepsilon][q^2 - \beta + i\varepsilon]}$$

$$= \frac{\pi^2}{i} \frac{d\mu}{p^2 - \mu + i\varepsilon} \left(\frac{a-\beta+\mu}{2}\right)^K a^l \left(\frac{\beta-a+\mu}{2}\right)^m \beta^n \frac{\sqrt{\Lambda(a,\beta,\mu)}}{\mu}. \quad (A.28)$$

2. The Boson Integral

We want to calculate the boson functions Q_i defined in Eq. (42) of the text. We will prove the following theorem:

The boson functions are related to the fermion function A^{II} with the massless Green's functions by the formulae

$$Q_0(s; a, \beta) = \left(1 - s\frac{\partial}{\partial s}\right) Q_g, \quad Q_1(s; a, \beta) = \left(1 + s\frac{\partial}{\partial s}\right) Q_g, \quad Q_2(s; a, \beta) = \left(2 + s\frac{\partial}{\partial s}\right) Q_g,$$

$$Q_g(s; a, \beta) = \frac{2}{s} A^{II}(s; a, \beta) = 2 b(s; a, \beta). \quad (A.29)$$

The proof is given in the last part of the appendix, where we state a more general theorem.

Of course we do not calculate Q_g with Eq. (A.29), but calculate Q_g directly and use (A.29) to determine the more complicated twofold convolution A^{II}. The explicit formula for A^{II} is given in Appendix 1 (A.13). The derivative b' is

$$\frac{\partial b(s; a, \beta)}{\partial s} = \frac{1}{a\beta}\left\{-\frac{2a\beta}{3s^2} + \frac{1}{6}\ln\frac{-s}{\sqrt{a\beta}} + \left(\frac{a^2-\beta^2}{2s^2} - \frac{(a-\beta)^2}{3s^3}\right)\right\}\ln\sqrt{\frac{a}{\beta}}$$

$$- \frac{(2a+s)(a-s)^2}{6s^3}\ln\left(1 - \frac{s}{a}\right) - \frac{(2\beta+s)(\beta-s)^2}{6s^3}\ln\left(1 - \frac{s}{\beta}\right) \quad (A.30)$$

$$- \frac{1}{12}\left(\frac{1}{s} + \frac{a+\beta}{s^2} - 2\frac{(a-\beta)^2}{s^3}\right) R(s; a, \beta).$$

The Q_i functions are linear combinations of (A.13) and (A.30). For equal masses one obtains (A.18) and (45) − (47). The lowest terms of the asymptotic expansions of b (for small resp. large s) are

$$b(s; a, \beta) = \frac{\ln a - \ln \beta}{2(\beta - a)} + \frac{s}{6 a \beta} \ln \left(\frac{-s}{\sqrt{a \beta}} \right)$$

$$+ \frac{s}{6 a \beta} \left[\frac{a^3 - 3 a^2 \beta - 3 a \beta^2 + \beta^3}{2(a - \beta)^3} \ln \frac{a}{\beta} + \frac{2 a \beta}{(a - \beta)^2} - \frac{11}{6} \right] + \dots, \quad (A.31)$$

$$b(s; a, \beta) = \frac{1}{s} \left[1 + \frac{a}{2 s} \ln \left(\frac{-s}{a} \right) + \frac{\beta}{2 s} \ln \left(\frac{-s}{\beta} \right) \right] + \dots . \quad (A.32)$$

3. Convolution with the Massless Green's Function

The Green's function G (1) of the pole spinor theory has a simple form in coordinate space, too.

$$\int e^{ipx} \frac{p^\mu}{(r^2 + i \varepsilon)^2} \, \mathrm{d}^4 x = \frac{i}{8 \pi^2} \frac{x^\mu}{x^2 - i \varepsilon} := G^\mu(x) . \quad (A.33)$$

From (A.33) we will deduce that convolutions with this massless dipole propagator can be replaced by one single integration: Multiplying (A.33) by x_μ we get a constant or the δ-function in momentum space. The convolution thus becomes trivial.

The multiplication by x_μ corresponds to the derivative $-i(\partial / \partial p^\mu)$ in momentum space. Inverting this differentiation we obtain the convolution with G by one integration. For the special case of the fermion integral we will write down this steps explicitly.

We consider the convolution of two propagators

$$\widetilde{p}^{\mu\nu}(I) := \int \frac{\mathrm{d}^4 p}{(2 \pi)^2} \, F_1^\mu(p) F_2^\nu(I - p) = g^{\mu\nu} P_g(I^2) + \frac{I^\mu I^\nu}{I^2} P_I(I^2)$$

with

$$\widetilde{F}_i^\mu(p) := \int \varrho_i(m^2) \frac{p^\mu}{(p^2 - m^2 + i \varepsilon)} \quad (A.34)$$

and the convolution of this $\widetilde{P}^\nu$ with the massless Green's function G of the pole theory

$$\widetilde{S}^{\mu\nu\lambda}(I) := \int \frac{\mathrm{d}^4 q}{(2 \pi)^4} \, \widetilde{P}^{\mu\nu}(q) G^\lambda(I - q) . \quad (A.35)$$

We will prove:

$$I^2 \widetilde{P}_g(I^2) = \frac{8 \pi^2}{i} \widetilde{S}_{\nu\lambda}^\nu I^\lambda . \quad (A.36)$$

The fouriertransforms of (A.24) resp. (A.35) are

$$P^{\mu\nu}(x) = F_1^\mu(x) \cdot F_2^\mu(x) , \quad (A.37)$$

$$S^{\mu\nu\lambda}(x) = P^{\mu\nu}(x) \cdot G^\lambda(x) \quad (A.38)$$

with

$$F_i^\mu(x) = \int \frac{\mathrm{d}^4 p}{(2 \pi)^4} \, e^{-ipx} \widetilde{F}_i^\mu(p) = : x^\mu f_i(x^2) . \quad (A.39)$$

The theorem (A.36) reads in x-space

$$- \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\gamma} P_g(x^2) = \frac{8 \pi^2}{i} i \frac{\partial}{\partial x_\mu} S_{\nu\mu}^\nu(x) . \quad (A.40)$$

Proof:

From (A.34) we isolate $\widetilde{P}_g(I^2)$

$$\widetilde{P}_g(I^2) = \frac{1}{3}\left(g_{\mu\nu} - \frac{I_\mu I_\nu}{I^2}\right)\widetilde{P^{\mu\nu}}(I) \tag{A.41}$$

and obtain for the left hand side of (A.40)

$$-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu}P_g(x^2) = \frac{1}{3}\left[-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu}x^\nu f_1(x^2)x_\nu f_2(x^2) + \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}x_\mu f_1(x^2)x_\nu f_2(x^2)\right]$$

$$= \frac{\partial}{\partial x_\mu}\left[x_\mu f_1(x^2)f_2(x^2)\right]. \tag{A.42}$$

The right hand side yields

$$8\pi\frac{\partial}{\partial x_\mu}S_{\nu\mu}{}^\nu(x) = \frac{\partial}{\partial x_\mu}\left[x^\nu f_1(x^2)x_\nu f_2(x^2)\frac{x_\mu}{x^2}\right]. \tag{A.43}$$

Therefore we have proved (A.40), and (A.36) after fouriertransformation. Replacing the general propagators F_i by the special ones of the pole spinor theory and introducing the appropriate factors of A^{II} (18) and Q_i (42), we obtain Eq. (A.29) of Appendix 2.

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